TIME-VARYING EXCHANGE RATE EXPOSURE COEFFICIENTS (EXPOSURE BETAS): EVIDENCE FROM COUNTRY LEVEL STOCK RETURNS

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ABSTRACT

This paper uses time-varying second moments to investigate exchange rate exposure betas. Using a BEKK-GARCH(1,2,1)-M model, time-varying exchange rate exposure betas are obtained with explicit focus on the non-orthogonality between exchange rate changes and market returns. We look into certain aspects of the stochastic structure underlying the exposure betas. An important finding of the paper is that, although exchange rate exposure betas are likely to vary over time, they follow mean-reverting long-memory processes. The presence of mean-reverting exchange rate exposure coefficients has important implications for investment and hedging strategies.

Key words: Time-varying exchange rate exposure; Multivariate GARCH-M models; International CAPM; Fractionally integrated processes

JEL Classification: C22; F31; F37; G12; G15

1. INTRODUCTION

Most of the studies in exchange rate exposure literature implicitly assume that the exposure coefficients remain unchanged over time. Nevertheless, there are several reasons to assume that exposure coefficients are time-varying. First, a country’s composition and/or shares of exports and imports may change drastically over time due to both external and internal factors. Changes in demand due to the rise of new competitors in international arena is an example for the former while the introduction of trade liberalization policies is an example for the latter. Second, as substitutes are being introduced, elasticity of demand for a country’s products and competitive structure of industries are likely to change over time. Allayannis and Ihrig (2001) argue that the changes in competitive structure affect the exchange rate exposure of industries. Third, financial market deregulations and liberalization attempts may lead to changes in foreign investments in local financial assets which in turn affect the exchange rate exposure of a country’s stocks.

Fourth, the change in location of production of MNCs in response to persistent strong currency positions may lead to the changes in the sensitivity of sectoral returns to exchange rate changes. Fifth, incidents like the 1997 currency crisis may lead to remarkable volatility changes in exchange rate markets.

In exchange rate exposure literature, there are three groups of studies that analyze the time-varying nature of exposure betas. The first group uses some primitive methods for this purpose. For instance, Dominguez and Tesar (2006) divide the sample period into a few sub-periods and estimate an exposure coefficient for each sub-period. Williamson (2001) uses dummy variables to distinguish between sub-periods and observe that the exposure is time-variant. Alternatively, Entoff and Jamin (2003) use overlapping moving window regressions to show the time-varying behaviour of the exchange rate exposure of a bunch of German firms. Bodner and Wong (2003) also use moving window regressions with various return horizons (1, 3, 6 and 12 months).

The second group of studies uses pre-specified determinants of exposure coefficients to analyze the time-variation of exposure. Allayannis (1997) suggests that exposure beta is determined by export and import shares.

a. Allayannis (1997) observes that the status of some US industries change from net exporters to net importers within the same sample period.

Using an appropriate model to accommodate this relationship, the study cites evidence for time-variation of exposure in some 4 digit level SIC industries. He reports that the same data set at industry level show significant exchange rate exposure only when the exposure is assumed to be time-varying. Allayannis and Ihrig (2001) inquire into the same phenomenon in terms of three determinants of exposure: (a) an industry’s competitive structure where it sells its production; (b) the interaction of the competitive structure of the export market and the export share; (c) the interaction of the competitive structure of the imported input market and its imported input share. Mark-ups are used as a measure of the competitive structure. Bodner et al. (2002) suggest a somewhat similar model in terms of time-varying exchange rate pass-through, though they are not able to show significant evidence for time-varying exposure. Chiao and Hung (2000) use the same determinants appearing in Allayannis (1997) to examine the time variation in the exchange rate exposure of Taiwanese exporting firms. In addition, they employ dummy variables to check whether the exchange rate exposure is affected by the timing of three liberalization effects introduced within the economy. Bodnar and Gentry (1993) add a few more factors to the list of pre-specified determinants of exposure, namely, whether the relevant industry produces traded or non-traded products, the amount of internationally-priced inputs used and the industry’s foreign direct investment. In an attempt of seeking the determinants of the exposure of Japanese firms, Chow and Chen (1998) use three proxies for the hedging incentives which in turn depend on the firm size. In addition to aggregate export and import shares to GDP, Entorf and Jamin (2003) use the absolute distance between exchange rates and their long-run mean as a determinant of exposure.

The third group of studies, which include Hunter (2005) and Lim (2005), use time-varying second moments to derive time-varying exchange rate exposure betas. While the former study analyzes time-varying exchange rate exposure of small and large firms using Fama-French-type size-based portfolios the latter derives time series of both market and exposure betas at country level. More importantly, Lim allows for non-orthogonality between the factors, a feature that Hunter fails to accommodate.

The contribution of the second group of studies to the literature is more appealing than that of the first group as they also show the determinants of the time-variation in exposure. These include factors like time-varying export and import shares, mark-ups and pass-through. Nevertheless, those studies are not without limitations. First, the studies that analyze the time-variation in exchange rate exposure in terms of a set of pre-specified variables implicitly rely on a somewhat questionable assumption that there are no other (left out) determinants of time-variation. However, mainly due to the absence of theoretical explanations of such relationships, there may be unidentified factors which are yet important in explaining the time-variation in exposure. Second, the unavailability of data for much shorter return horizons may force the researchers to ignore some determinants or use unsuitable proxies. Moreover, if return horizon in question is a day, any of such data series is not available on daily basis. Third, the above studies seem to have neglected the impact of the time-varying volatilities which is one of the major and crucial determinants of model parameters and the time-varying element of them. Fourth, the underlying stochastic structure of the exchange rate exposure betas is largely left unexamined. For instance, such studies do not answer the question whether the time-varying exposure betas are mean-reverting.

This paper uses time-varying second moments to investigate exchange rate exposure betas. However, unlike Hunter (2005) or Lim (2005), we directly use the mean structure of conditional ICAPM theorized by Adler and Dumas (1983) and make econometrically feasible by De Santis and Gerard (1998) to derive time-varying exposure betas. We look into certain aspects of the underlying stochastic structure of exposure betas. Time-varying beta series are also used in some applications. An important empirical finding of the paper is that, although exchange rate exposure betas are likely to vary over time, they follow mean-reverting long-memory processes. The presence of mean-reverting exchange rate exposure coefficients has important implications for investment and hedging strategies.

The rest of this paper is organized as follows. The conceptual framework of the study that is based on a conditional international CAPM is outlined in section 2. Section 3 presents the information related to data and a preliminary analysis of the returns and exchange rate series. In Section 4, we report the main empirical findings. In addition to the estimation results, this section includes an inquiry into the stochastic structure of time-varying exposure beta, a comparison of exposure among countries using the concept of stochastic dominance and an analysis of time-varying currency premiums. Some concluding remarks are included in Section 5.

2 CONCEPTUAL FRAMEWORK OF THE ANALYSIS

According to Adler and Dumas (1983), investors living in a world with purchasing power parity (PPP) violations, which is “the rule rather than exception”, usually think of hedging against the purchasing power risk that would stem from unexpected inflation. The asset holding of a representative investor in such a context is characterized by two types of portfolios: (a) a

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3 For instance, Allayannis and Ihrig (2001) make an assumption that mark-ups vary on annual basis, though they work with monthly data, due to the unavailability of mark-ups on monthly basis.
world market portfolio of risky assets; and (b) “a personalized hedge portfolio which constitutes the best protection against inflation as [the investor] perceives it”. As such, the expected return on an asset may consist of two parts: the market premium which depends on the asset’s world market risk and an additional premium which depends on its usefulness to hedge purchasing power risk. Assuming a world with \( L + 1 \) number of countries (and currencies), the expected excess return on equity is formally expressed as:

\[
E_{t-1}(r_{it}) = \lambda_{m,t-1} \cdot \text{Cov}_{t-1}(r_{it}, r_{mt}) + \sum_{l=1}^{L} \lambda_{\pi,t-1} \cdot \text{Cov}_{t-1}(r_{lt}, \pi_{lt})
\]

where \( \lambda_{m,t-1} = \theta_{t-1} \cdot \frac{1}{\sum_{l=1}^{L} W_{lt,t-1} \cdot \frac{1}{\theta_{l}}} \) and \( \lambda_{\pi,t-1} = \theta_{t-1} \left( \frac{1}{\theta_{l}} - 1 \right) \frac{W_{lt,t-1}}{W_{t-1}} \).

In Equation 1, \( E_{t-1}(\cdot) \) and \( \text{Cov}_{t-1}(\cdot, \cdot) \) are expectations and covariances conditional on the current information set \( I_{t-1} \); \( r_{it} \) is excess return on a certain asset \( i \); \( r_{mt} \) is excess return on world market portfolio; \( \pi_{lt} \) is the inflation rate in country \( l \); \( \theta_{l} \) is coefficient of relative risk aversion for investors from country \( l \); \( \theta_{t-1} \) is average risk aversion coefficient for each country weighted by relative wealth \( \frac{W_{lt,t-1}}{W_{t-1}} \); \( W_{t-1} \) is the aggregated wealth. The conditional covariance between \( r_{lt} \) and \( r_{mt} \) represents the world market risk and, as in the case of standard CAPM, \( \lambda_{m,t-1} \) is known as the market price of risk. The conditional covariances between \( r_{lt} \) and \( \pi_{lt} \) represent both inflation and currency risk that stem from PPP violations. Specifically, \( \lambda_{\pi,t-1} \cdot \text{Cov}_{t-1}(r_{lt}, \pi_{lt}) \) is the inflation premium that the investor demands for the co-movement between the asset’s nominal return and the inflation in the \( l \)th country.

As most practitioners do, this model can be simplified by making the assumption that inflation in a certain country is non-stochastic[^4] (see Dumas and Solnik (1995), De Santis and Gerard (1997)). In such a world, PPP deviations are precisely reflected in exchange rate changes. This is a plausible assumption given the fact that the fluctuations of inflation are negligible as compared to exchange rate fluctuations (Cappiello et al. 2003). Then, the only random component in \( \pi_{x,t} \) is currency risk and the changes in PPP deviations are identified with exchange rate changes. Accordingly, \( \lambda_{x,t-1} \) can be denoted as \( \lambda_{x,t-1} \) and identified as the currency price of risk associated with the currency in the \( \theta \)th country. Thus, seen, Equation 1 consists of \( L \) number of currency premiums \( \sum_{l=1}^{L} \lambda_{\pi,t-1} \cdot \text{Cov}_{t-1}(r_{lt}, r_{st}) \) that stem from the covariances between the asset’s returns and the changes in the exchange rates in \( L \) number of other countries. Empirically, this large number of currency premiums is limited to a few (see De Santis and Gerard, 1998, among others).

Assuming a single currency factor, the ICAPM relationship represented by Equation 1 can also be expressed as follows:

\[
E_{t-1}(r_{it}) = \beta_{m,t-1} \cdot E_{t-1}(r_{mt}) + \beta_{x,t-1} \cdot E_{t-1}(r_{st}).
\]

where \( \beta_{m,t-1} \) and \( \beta_{x,t-1} \) are market beta and the exchange rate exposure beta, respectively. \( \beta_{m,t-1} \) measures the asset’s exposure to market risk while \( \beta_{x,t-1} \) measures its exposure to currency risk. Viewed from this perspective, the time-varying nature of the second moments makes both betas time-varying. The intuition is that, while the expected returns on an asset is proportional to market returns and exchange rate changes, depending on the conditioning information that is publicly available at time \( t-1 \), the proportionality factors (market and exchange rate exposure betas) themselves are also time-varying. In other words, the investors are sensitive to “the new information that periodically becomes available to [them], who then use it to adjust their investment strategies” (Harvey, 2001).

Within the above framework, we take the viewpoint of a US investor who wants to invest in foreign as well as US assets in order to hedge against currency risk. Return on the relevant country stock index is assumed to be a reasonable proxy for the return on an asset in that country. Each country is considered separately forming a pair with the US. The main reason for this arrangement is the parsimony associated with it. Viewed from this perspective, the expected return equation for each country index includes only one bilateral exchange rate between the US dollar and the relevant currency. Obviously, this is an incomplete specification of Adler and Dumas (1983) model in the sense that there must be a number of other currency premiums in the expected return equation. However, we may neglect such an information loss as our main objective is to derive time-

[^4]: If inflation is stochastic then the model can be expressed in such a way that the expected returns are dependent on three premiums, namely, market, currency and inflation. See Moerman and van Dijk (2006), for such a variation of the model.
varying exchange rate exposure betas, but not to test the
validity of this version of ICAPM\(^5\). Since the returns on
assets in each country is evaluated with respect to the
changes in the exchange rate with the US dollar, the
suggested structure offers a common yard stick with
which the exposure to currency risk in each country can
be compared.

Assuming that the market returns and exchange rate
changes are not necessarily orthogonal, we suggest the
following parsimonious version of ICAPM-related mean
structure for the purpose of deriving time-varying
exchange rate exposure betas.

\[
E_{t-1}(r_{i,t}) = \lambda_{X,i-1} \text{Cov}_{t-1}(r_{i,t}, r_{x,t}) + \lambda_{M,i-1} \text{Cov}_{t-1}(r_{i,t}, r_{m,t})
\]

\( (3) \)

\[
E_{t-1}(r_{x,t}) = \lambda_{X,i-1} \text{Var}_{t-1}(r_{x,t}) + \lambda_{M,i-1} \text{Cov}_{t-1}(r_{x,t}, r_{m,t})
\]

\( (4) \)

\[
E_{t-1}(r_{m,t}) = \lambda_{X,i-1} \text{Cov}_{t-1}(r_{m,t}, r_{x,t}) + \lambda_{M,i-1} \text{Var}_{t-1}(r_{m,t})
\]

\( (5) \)

where \( r_{i,t} \) is return on country \( i \)’s stock index at time
\( t \); \( r_{m,t} \) is return on the world market portfolio at time
\( t \); \( r_{x,t} \) is the change in bilateral nominal exchange rate
between the US dollar and the currency of country \( i \) at
time \( t \); \( \lambda_{M,i-1} \) is market price of risk; and \( \lambda_{X,i-1} \) is the
currency price of risk. Since we allow for non-
orthogonality between market returns and exchange rate
changes, a non-zero \( \text{Cov}_{t-1}(r_{m,t}, r_{x,t}) \) term enters
into mean equations 4 and 5. The exchange rate is
expressed as the US dollar price of foreign currency and
an increase implies a depreciation of US dollar relative
to the relevant currency. As for the US, a trade-weighted
exchange rate is expressed as the US dollar price of
foreign currency.

Contrary to the common practice, we do not
convert the returns into a common/reference currency.
Returns on each country index are measured in the
relevant local currency. We also select a value-weighted
world market index which is not converted into a
common/reference currency. The purpose of this
exercise is to obtain country level portfolios and a world
market portfolio which represent “the theoretical
performance of an index without any impact from
foreign exchange fluctuations” (MSCI, 1998). Our
reluctance to convert returns on country indexes and the
world market index into a common currency is due to a
few theoretical and empirical reasons. First, it helps us
separate market risk from currency risk. As
Giannopoulos (1995) argues, these two risks are not
additive and conversion of various country stock index
returns into a common currency will have an adverse
impact on their volatility. Second, conversion of country
index returns (the dependent variable) into US dollars
using the exchange rate chosen would lead to inaccurate
exposure coefficients because the changes in the same
exchange rate is an independent variable in the
regression\(^6\). Third, conversion of the returns on a world
market index denominated in a common currency
(mostly in US dollar) into local currency might have
resulted in an unacceptable degree of multicollinearity
between the two regressors. In addition to the resultant
inefficient parameter estimates, it would also lead to
unrealistic estimates of exchange rate exposure beta.

3 ECONOMETRIC METHODOLOGY

In order to derive time-varying exchange rate exposure
betas with time-varying second moments, we turn to
multivariate GARCH-type models. More specifically,
assuming constant prices for market risk and currency
risk\(^7\) and that the market returns and exchange rate
changes are not necessarily orthogonal, we employ a
trivariate BEKK-GARCH (\( p, q, K \))-M model.

\[
r_{j,t} = \lambda_{0,j} + \lambda_{X,j} r_{X,t} + \lambda_{M,j} r_{M,t} + \theta_{j} \varepsilon_{j,t-1} + \varepsilon_{j,t}
\]

\( j = i, m, x \)

\( (6) \)

\[
z_{t} = \varepsilon_{t} H_{t}^{-1/2}
\]

\( (7) \)

\[
\varepsilon_{t} | I_{t-1} = (\varepsilon_{i,t}, \varepsilon_{m,t}, \varepsilon_{x,t})' | I_{t-1} \sim N(0, H_{t})
\]

\( (8) \)

\[
H_{t} = C'C + \sum_{k=1}^{K} \sum_{l=1}^{q} A_{kl} \varepsilon_{k,t-1} \varepsilon_{l,t-1}' A_{kl} + \sum_{k=1}^{K} \sum_{l=1}^{p} B_{kn} H_{t-n} B_{kn}
\]

\( (9) \)

\[
B = [H_{t}^{-1} H_{t}]^{-1} H_{t}
\]

\( (10) \)

where \( r_{j,t} \) is a 3 x 1 vector that consists of three elements:
return on country index at time \( t \) (\( r_{i,t} \)), return on world

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\(^5\) See De Santis and Gerard (1998) and Cappiello et al
(2003), for attempts to test the validity of ICAPM using
a set of countries and a number of relevant exchange
rates.

\(^6\) Strictly speaking, this exercise is done using the
exchange rates moderated by the base year rate.
However, there exists a strong correlation between a
series converted using the moderated exchange rates and
a series converted using the current rates.

\(^7\) Constant prices can be justified on the grounds that the
suggested model is just a data generating process to
test the time-varying nature of exchange rate exposure
betas in terms of time-varying second moments.
market portfolio at time \( t \) ( \( r_{m,t} \) ) and changes in bilateral nominal exchange rate between the US dollar and the currency of country \( i \) at time \( t \) ( \( r_{x,i,t} \))^8. An intercept and a MA(1) term is included in each of the three mean equations in order to capture any remaining risk or market inefficiencies. \( \epsilon_t \) is a \( 3 \times 1 \) vector of residuals from mean equations in (6), which are assumed to be normally distributed with mean 0 and variance \( H_{t} \), which is \( 3 \times 3 \) variance covariance matrix. And, \( \epsilon_t \mid I_{t-1} \) denotes the vector of random shocks at time \( t \) given all available information at time (\( t-1 \)). In addition, \( h_{x,j} \) and \( h_{M,j} \) are \( 3 \times 1 \) vectors that consist of the elements in the second and the third columns of \( H_{t} \), respectively^9. Finally, \( z_t \) denotes the standardized residuals that are assumed to be identically and independently distributed with mean 0 and variance 1.

Parameters \( \lambda_M \) and \( \lambda_X \) are market price of risk and currency price of risk, respectively. \( C \) is an upper triangular \( 3 \times 3 \) matrix that contains the constants in conditional variance and covariance equations. Both \( A_{k,j} \) and \( B_{k,n} \) are \( 3 \times 3 \) parameter matrixes. Although the use of a trivariate model offers the opportunity to capture the interdependence between the volatilities in terms of non-zero off-diagonal terms in parameter matrices, we make the restrictive assumption that parameter matrices \( A_{k,j} \) and \( B_{k,n} \) are diagonal for two reasons. First, the full BEKK formulation is less parsimonious and computationally tedious^10. Second, as the results of diagnostic tests reported in Section 4 show, the suggested diagonal version of the model sufficiently captures the non-linearities in stock returns and exchange rate changes. For parsimony in the suggested BEKK model, we set \( K = 1 \). As a residual analysis based on Ljung-Box statistic reveals, the optimal lag orders for GARCH and ARCH terms are as follows: \( p = 1, q = 2 \).

Following Lim (2005), time-varying betas can be obtained through Equations 9 and 10. In Equation 9, \( H_{t}^{mv} \), \( H_{t}^{vy} \) and \( H_{t}^{wu} \) are the conditional variance-covariance matrixes of the assets to be priced, the factors with which the assets are priced, and between the assets and factors, respectively.

Assuming that the standardized residuals of the suggested trivariate GARCH model are conditionally normally distributed, the conditional log-likelihood of residual vector \( \epsilon_t \) at time \( t \) can be defined as follows:

\[
\ell(\phi) = -\frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln|H_t| - \frac{1}{2} \epsilon_t'H_t^{-1}\epsilon_t, \quad (11)
\]

The log-likelihood function of the sample is obtained as \( L(\phi) = \sum_{t=1}^{T} \ell(\phi) \), where \( T \) is the number of observations. The parameter vector \( \theta \) of the trivariate BEKK-GARCH(1,2,1)-M model is estimated by maximizing \( L \) with respect to \( \phi \). In order to accommodate the non-normal features reflected in the basic statistics of country returns and the exchange rate changes, all estimates of the parameters are obtained through the quasi-maximum likelihood (QML) estimation method proposed by Bollerslev and Wooldridge (1992). Under certain regularity conditions, the QML estimate is assumed to be consistent and asymptotically normal. Therefore, statistical inference can be drawn due to robust standard errors. Required computer programs are coded in GAUSS and use BHHH algorithm to compute QML estimates.

### 4 DATA

We use a sample of nine countries: the US, UK, Canada, Japan, Australia, Korea, Singapore, Taiwan and Thailand. The sample is assumed to represent a balanced combination of developed and emerging markets. We use daily closing stock prices for the period from 5th Jan 1999 to 30th Dec 2005^11. The resultant sample period provides us with 1824 observations. Except for the exchange rate used for the US, all data series are from Morgan Stanley Capital International (MSCI) and extracted from Datastream. Country level portfolios are represented by MSCI country indexes measured in relevant local currency. World market portfolio is represented by the MSCI world market index MSWRLDL. It is a value-weighted world market index which is not converted into a common/reference currency and, therefore, free from exchange rate fluctuations (Giannopoulos, 1995; MSCI, 1998). Exchange rates used for non-US countries are MSCI bilateral rates that show the units of the relevant currency per one US dollar. The rates are inverted to

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8 The exchange rate is expressed as the US dollar price of foreign currency and an increase implies a depreciation of US dollar relative to the relevant currency. As for the US, a trade-weighted exchange rate is expressed as the US dollar price of foreign currency.

9 Since we allow for non-orthogonality between market returns and exchange rate changes, a non-zero \( h_{xm,t} \) term enters into mean equations for market returns and exchange rate changes.

10 In our initial round of regressions, we found that the full BEKK model did not converge in some cases.

11 The currency crisis period is excluded from the sample in order to avoid the impact of unusual currency moments.
express the exchange rates as the US dollar price of the foreign currency. A trade-weighted exchange rate compiled by the Bank of England is used to measure the exposure of the US assets.

Continuously compounded daily returns and exchange rate changes are calculated as follows:

\[ r_{j,t} = \ln \left( \frac{R_{j,t}}{R_{j,t-1}} \right) \times 100 \quad j = i, m, x \]

where \( R_{j,t} \) and \( R_{j,t-1} \) are the closing values of stock prices/exchange rates for the trading days \( t \) and \( (t-1) \) respectively.

All return series show excess kurtosis which ranges from the lowest 1.773 (Japan) to the highest 7.06 (Thailand). Jarque-Bera statistic is extremely high in all cases. Exchange rate changes are less skewed than the returns and show somewhat lower excess kurtosis. Except for Taiwan and Thailand, excess kurtosis of the exchange rate changes is lower than that of the returns on the relevant country index. The high Jarque-Bera statistic together with excess kurtosis in some cases implies that the exchange rate changes are not normally distributed. The non-normal features of both country stock returns and exchange rate changes justify the use of QML method of estimation.

As evidenced by the augmented Dicky-Fuller test, continuously compounded returns on all country indexes and the world market index and exchange rate changes are stationary. The Ljung-Box test for returns evaluated at 20 lags (Q(20)) reveals that returns for Canada, Korea, Taiwan, Thailand, UK and World market are not free from linear dependencies. However, except for Taiwan and Thailand, exchange rate changes in all the other cases do not show linear dependencies. In addition, the Ljung-Box test for squared returns evaluated for 20 lags (Q^2(20)) displays that all return and exchange rate series possess a great deal of non-linear dependencies. This provides some empirical support for the use of GARCH-type models to derive time-varying exchange rate exposure betas.

5 EMPIRICAL FINDINGS

In this section, we first report the results of a few tests that show the likeliness of time-varying or unstable parameters in the selected sample. After this pre-estimation assessment, we move onto deriving time-varying exchange rate exposure betas and some diagnostic checks for adequacy of the proposed model to derive exposure betas. This is followed by a brief investigation of the stochastic structure of time-varying exposure betas. Finally, time-varying exposure beta series derived in the previous sub-sections are used in two applications.

5.1 Some pre-estimation results

We use a battery of tests to show that countries selected in the sample are more likely to possess time-variant (unstable) exchange rate exposure betas. All tests are based on OLS estimation of the conventional augmented market model that is widely used to estimate exchange rate exposure. The first such test is the cumulative sum of squared recursive residuals (CSSRR) test suggested by Brown et al. (1975). The CSSRR test is performed at the 5% level of significance. During the sample period, the CSSRR crosses the critical value boundaries in all cases, thus suggesting the underlying parameter instabilities. White’s (1980) test for unconditional heteroskedasticity and ARCH-LM test for conditional heteroskedasticity are used to diagnose possible parameter instabilities. White’s test statistic is significant in all cases except for Thailand at 5 degrees of freedom at the significance level 5%, suggesting the presence of unconditional heteroskedasticity. ARCH-LM test statistic for 5 lags is significant for all the cases at the significance level 5%. Results from all three tests suggest that the constant parameters in the specification represented by Equation 2 are highly likely to be unstable.

5.2 Time-varying exchange rate exposure betas

The maximum likelihood estimates for the suggested trivariate BEKK-GARCH(1,2,1)-M model are reported in Table 1. According to ICAPM reasoning, the market price of risk (\( \lambda_M \)) must be positive and the same for all countries. However, there is no such restriction for the currency price of risk (\( \lambda_X \)). As the estimation results reported in Panel A indicate, \( \lambda_M \) is positive and does not vastly vary across countries. More specifically, it ranges from 0.0237 (Canada) to 0.0573 (US), and is not statistically significant in any of those cases. Unlike market price of risk, \( \lambda_X \) varies remarkably across countries between the range -0.4539 (UK) and 0.0405 (Canada). Since the relevant exchange rate varies across countries, this variation in the parameter can be understood. In all cases, currency price of risk is also not statistically significant.

12 Results are not shown here.

13 This refers to the constant parameter version of the regression equation in (2).

14 To conserve space, results are not shown here.

15 These results are consistent with the previous findings in the literature. For instance, De Santis and Gerard (1998) and Cappiello et al. (2003) also find that both
Table 1: Maximum likelihood estimates for the trivariate diagonal BEKK GARCH(1,2,1)–M model

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Aust</th>
<th>Canada</th>
<th>Japan</th>
<th>Korea</th>
<th>Sing</th>
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<td>0.0346</td>
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</tr>
<tr>
<td>$b_t$</td>
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<td>0.9765*</td>
<td>0.9617*</td>
<td>0.9423*</td>
<td>0.9748*</td>
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<td>(32.68)</td>
</tr>
<tr>
<td>$a_t$</td>
<td>0.1833*</td>
<td>0.2083*</td>
<td>0.2427*</td>
<td>0.1633*</td>
<td>0.2603*</td>
</tr>
<tr>
<td></td>
<td>(3.40)</td>
<td>(7.28)</td>
<td>(8.93)</td>
<td>(6.98)</td>
<td>(2.31)</td>
</tr>
<tr>
<td>$d_t$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.1700</td>
</tr>
<tr>
<td>$b_s$</td>
<td>0.9737*</td>
<td>0.9888*</td>
<td>0.9516*</td>
<td>0.9637*</td>
<td>0.9748*</td>
</tr>
<tr>
<td></td>
<td>(169.72)</td>
<td>(168.24)</td>
<td>(101.41)</td>
<td>(150.79)</td>
<td></td>
</tr>
<tr>
<td>$a_s$</td>
<td>0.1976*</td>
<td>0.3123*</td>
<td>0.2792*</td>
<td>0.2363*</td>
<td>0.2410*</td>
</tr>
<tr>
<td></td>
<td>(6.54)</td>
<td>(10.82)</td>
<td>(6.44)</td>
<td>(6.10)</td>
<td></td>
</tr>
<tr>
<td>$b_m$</td>
<td>0.9620*</td>
<td>0.9644*</td>
<td>0.9642*</td>
<td>0.9610*</td>
<td>0.9748*</td>
</tr>
<tr>
<td></td>
<td>(90.58)</td>
<td>(83.39)</td>
<td>(111.07)</td>
<td>(139.43)</td>
<td></td>
</tr>
<tr>
<td>$a_m$</td>
<td>0.0039</td>
<td>0.1114*</td>
<td>0.2080*</td>
<td>0.2410*</td>
<td>0.2410*</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(3.11)</td>
<td>(12.46)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_m$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.2603*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-0.75)</td>
</tr>
</tbody>
</table>

Notes: $t$-values are in parenthesis; * indicates the significance at %5 level.

Furthermore, all GARCH terms (denoted by $b_j$ for $j = i, m, x$) are highly significant, suggesting that the conditional variances are highly correlated to the past conditional variances. For Singapore, Taiwan and Thailand, an ARCH (2) term (denoted by $d_j$) is included in the variance equations of country returns. Except for three cases (Australia, Canada and Japan), an ARCH (2) term is also included in the variance equation of the world market returns. However, additional ARCH terms are not required for the variance equation of exchange rate changes of any country. At least one ARCH term is significant in all the cases (except for world market return for Singapore), suggesting the presence of volatility clustering in both stock and exchange rate markets of all countries.

The diagnostic checks for the estimated model, results of which are not shown here to conserve space, reveal that linear and non-linear dependencies have been adequately captured by the proposed trivariate BEKK GARCH(1,2,1)-M model. Ljung-Box statistics for standardized and squared standardized residuals evaluated for 20 lags ($Q(20)$ and $Q^2(20)$, respectively) are significantly low as compared to those of the return and exchange rate series. Except for a few cases, $Q(20)$ and $Q^2(20)$ are well below the critical value of 31.481 at the 5 % level. These results imply that the suggested model is adequate to derive reliable estimates of time-varying exchange rate exposure betas.

5.3 The stochastic structure of exchange rate exposure betas

Table 2 compares the mean values of time-varying exchange rate exposure betas and their OLS point estimates. The mean value of each series is reasonably close to the relevant OLS point estimate. Exposure beta associated with the bilateral exchange rate between the US dollar and the relevant currency is positive in seven cases and it is negative only for UK. Interestingly, the exposure beta of the US, which is associated with a trade-weighted exchange rate, is also positive. The intuition is that a US exporter can hedge against currency risk by investing only in UK assets, whose returns are negatively correlated with the depreciation of local currency. Importers or investors whose consumption basket consists of a lot of imported goods from the relevant countries can hedge against currency risk by investing in assets in any country except UK.

<table>
<thead>
<tr>
<th>Coeff</th>
<th>Taiwan</th>
<th>Thai</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{M}$</td>
<td>0.0486</td>
<td>0.0400</td>
<td>0.0345</td>
<td>0.0573</td>
</tr>
<tr>
<td></td>
<td>(1.06)</td>
<td>(0.80)</td>
<td>(0.70)</td>
<td>(1.05)</td>
</tr>
<tr>
<td>$\lambda_{X}$</td>
<td>-0.1658</td>
<td>-0.0848</td>
<td>-0.4539*</td>
<td>-0.1209</td>
</tr>
<tr>
<td></td>
<td>(-1.85)</td>
<td>(-1.05)</td>
<td>(-1.94)</td>
<td>(-0.56)</td>
</tr>
<tr>
<td>$b_t$</td>
<td>0.9737*</td>
<td>0.9888*</td>
<td>0.9516*</td>
<td>0.9637*</td>
</tr>
<tr>
<td></td>
<td>(169.72)</td>
<td>(168.24)</td>
<td>(101.41)</td>
<td>(150.79)</td>
</tr>
<tr>
<td>$d_t$</td>
<td>-0.0912</td>
<td>0.0481</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(-1.31)</td>
<td>(0.69)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_s$</td>
<td>0.8617*</td>
<td>0.9212*</td>
<td>0.9837*</td>
<td>0.9855*</td>
</tr>
<tr>
<td></td>
<td>(20.88)</td>
<td>(50.71)</td>
<td>(174.85)</td>
<td>(184.02)</td>
</tr>
<tr>
<td>$d_s$</td>
<td>-0.0039</td>
<td>0.1114*</td>
<td>0.2080*</td>
<td>0.2410*</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(3.11)</td>
<td>(12.46)</td>
<td></td>
</tr>
<tr>
<td>$b_m$</td>
<td>0.9620*</td>
<td>0.9644*</td>
<td>0.9642*</td>
<td>0.9610*</td>
</tr>
<tr>
<td></td>
<td>(90.58)</td>
<td>(83.39)</td>
<td>(111.07)</td>
<td>(139.43)</td>
</tr>
<tr>
<td>$d_m$</td>
<td>-0.2603*</td>
<td>-0.2250*</td>
<td>-0.1149*</td>
<td>-0.0447*</td>
</tr>
<tr>
<td></td>
<td>(-7.58)</td>
<td>(-4.79)</td>
<td>(-4.21)</td>
<td>(-4.23)</td>
</tr>
</tbody>
</table>

Notes: $t$-values are in parenthesis; * indicates the significance at %5 level.

16 It is worth making a special comment on two cases which do not satisfy this requirement: country returns for Thailand and exchange rate changes for Korea. As for the former, $Q(11)$ statistic is below the critical value up to 11 lags ($Q(11) = 13.71$). In the later, $Q^2(15)$ statistic is below the critical value up to 15 lags ($Q^2(15) = 14.51$).
The US dollar exchange rate is highly correlated with the returns on assets in Taiwan, Thailand and Korea.

Table 2: Comparison between OLS point estimates of betas and the mean values of time-varying betas

<table>
<thead>
<tr>
<th>Country</th>
<th>Market beta</th>
<th>Exposure beta</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS Mean of</td>
<td>OLS Mean of</td>
</tr>
<tr>
<td></td>
<td>$\beta_m$</td>
<td>$\beta_x$</td>
</tr>
<tr>
<td></td>
<td>$\beta_{m,t}$</td>
<td>$\beta_{x,t}$</td>
</tr>
<tr>
<td>Australia</td>
<td>0.1970</td>
<td>0.2158</td>
</tr>
<tr>
<td>Canada</td>
<td>0.8398</td>
<td>0.8661</td>
</tr>
<tr>
<td>Japan</td>
<td>0.4457</td>
<td>0.5443</td>
</tr>
<tr>
<td>Korea</td>
<td>0.5448</td>
<td>0.5243</td>
</tr>
<tr>
<td>Singapore</td>
<td>0.4179</td>
<td>0.3620</td>
</tr>
<tr>
<td>Taiwan</td>
<td>0.3579</td>
<td>0.3022</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.3149</td>
<td>0.2197</td>
</tr>
<tr>
<td>UK</td>
<td>0.8620</td>
<td>0.7993</td>
</tr>
<tr>
<td>US</td>
<td>1.1660</td>
<td>1.2044</td>
</tr>
</tbody>
</table>

Notes: Time-varying market and exchange rate exposure betas are obtained using the relationships

$$\beta_{m,t} = \frac{h_{m,t}h_{m,t} - h_{m,m}h_{m,t}}{h_{m,t}h_{m,t} - h_{m,m}}$$

and

$$\beta_{x,t} = \frac{h_{x,t}h_{x,t} - h_{x,m}h_{x,t}}{h_{x,t}h_{x,t} - h_{x,m}},$$

respectively.

Summary statistics of exchange rate exposure betas are reported in Panel A of Table 3. Standard deviation of exposure beta series ranges from 0.1156 for Australia to 1.4060 for Taiwan. Emerging markets like Korea, Taiwan, Singapore and Thailand have more volatile and higher exposure betas than those in the developed markets like the US, UK, Japan or Canada. Seven series are positively skewed whereas just two are negatively skewed. All exposure beta series are leptokurtic.

Table 3

Panel A: Preliminary statistics of time-varying exchange rate exposure betas

<table>
<thead>
<tr>
<th>Aus</th>
<th>Canada</th>
<th>Japan</th>
<th>Korea</th>
<th>Sing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.1049</td>
<td>0.0272</td>
<td>0.0792</td>
<td>0.8661</td>
</tr>
<tr>
<td>Max</td>
<td>0.7151</td>
<td>1.2227</td>
<td>0.8646</td>
<td>4.8935</td>
</tr>
<tr>
<td>Min</td>
<td>-0.1946</td>
<td>-0.8770</td>
<td>-0.7595</td>
<td>-3.5040</td>
</tr>
<tr>
<td>S D</td>
<td>0.1156</td>
<td>0.2487</td>
<td>0.1952</td>
<td>1.2186</td>
</tr>
<tr>
<td>Skew</td>
<td>1.1676</td>
<td>1.2890</td>
<td>0.0432</td>
<td>0.4595</td>
</tr>
<tr>
<td>Kurto</td>
<td>6.9273</td>
<td>11.0618</td>
<td>4.6935</td>
<td>3.9570</td>
</tr>
<tr>
<td>J-B stat</td>
<td>1584.93</td>
<td>5438.64</td>
<td>218.28</td>
<td>133.64</td>
</tr>
</tbody>
</table>

Summary statistics of exchange rate exposure betas are obtained using the relationships

$$\beta_{m,t} = \frac{h_{m,t}h_{m,t} - h_{m,m}h_{m,t}}{h_{m,t}h_{m,t} - h_{m,m}}$$

and

$$\beta_{x,t} = \frac{h_{x,t}h_{x,t} - h_{x,m}h_{x,t}}{h_{x,t}h_{x,t} - h_{x,m}},$$

respectively.

Panel B: Preliminary statistics of time-varying market betas

<table>
<thead>
<tr>
<th>Aus</th>
<th>Canada</th>
<th>Japan</th>
<th>Korea</th>
<th>Sing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.2142</td>
<td>0.8664</td>
<td>0.5445</td>
<td>0.5039</td>
</tr>
<tr>
<td>Max</td>
<td>0.8762</td>
<td>1.8926</td>
<td>1.3883</td>
<td>1.2852</td>
</tr>
<tr>
<td>Min</td>
<td>-0.0366</td>
<td>0.3811</td>
<td>-0.2390</td>
<td>-0.3974</td>
</tr>
<tr>
<td>S D</td>
<td>0.1264</td>
<td>0.2578</td>
<td>0.2573</td>
<td>0.2590</td>
</tr>
<tr>
<td>Skew</td>
<td>0.9869</td>
<td>0.9417</td>
<td>0.3113</td>
<td>0.4304</td>
</tr>
<tr>
<td>Kurto</td>
<td>4.8983</td>
<td>4.3717</td>
<td>2.9397</td>
<td>2.4022</td>
</tr>
<tr>
<td>J-B stat</td>
<td>563.38</td>
<td>412.15</td>
<td>29.70</td>
<td>83.39</td>
</tr>
</tbody>
</table>

Panel A and B in Table 3 indicate that, except for Australia, Canada and Japan, the standard deviation of exchange rate exposure beta is usually higher than that of market beta (irrespective of whether the economy is developed or emerging, market beta in each case is relatively less volatile). This difference is remarkably high for countries like Korea, Singapore, Taiwan and Thailand. Kurtosis of each exposure beta distribution is always higher than the kurtosis of its counterpart market beta distribution. This suggests that an exposure beta distribution tends to have more outliers than the outliers in its counterpart market beta distribution.

Figure 1 provides a visual glimpse of all time-varying exposure betas. As we use daily data, the estimates of time-varying betas may be “still volatile and inevitably subject to estimation error” (De Santis and Gerard, 1998). As such, the Hodrick-Prescott filtered trends of betas are also included in Figures 1. Exposure beta of Taiwan, Korea and Thailand fluctuates within a wide range whereas the exposure beta of Australia, Canada,
Japan, UK, and the US display somewhat meager fluctuations.

Next, we examine whether the exchange rate exposure betas are mean-reverting and stationary. Widely used semi-nonparametric Gewek and Porter-Hudak (1983) test is employed for this purpose\(^{17}\). In order to see the sensitivity of the estimates of the fractional difference parameter \(d\) to the choice of \(\alpha\), three values of \(\alpha\) are used here: 0.50, 0.55 and 0.60. First, we perform a one-sided test to check the validity of the null hypothesis of \(d = 0\) against the alternative of \(d > 0\). The null is

\[\ln I(\omega_s) = c + \phi \ln(4\sin^2(\omega_s/2)) + \zeta \quad \text{for} \quad s = 1, 2, \ldots, n(T)\]

where \(T\) is the number of observations in the series concerned; \(I(\omega_s)\) is the periodogram of a series at harmonic frequency \(\omega_s = (2\pi s/T)\) with \(s = 1, 2, \ldots, T - 1\); \(\zeta\) is random error; \(n\) represents the number of low frequency ordinates and is usually determined as \(n = T^{\alpha}\). OLS estimation of \(\phi\) provides a consistent estimate of \(-d\) in the ARFIMA process

\[d \ln(1 - \rho) = \phi \ln(1 - \rho) + \zeta\]

\(^{17}\)The test is based on the following spectral regression equation:
rejected at the 5% level for all cases except for Japan and Taiwan when $\alpha = 0.5$ (results are not reported), suggesting that all exposure beta series are more likely to be represented by an ARFIMA process. Then a second one-sided test is performed for the null hypothesis of $d = 1$ against the alternative of $d < 1$. Test results for exchange rate exposure beta are reported in Table 4. The null is rejected at the 5% level for all nine cases under all three $\alpha$ values except for Korea. Even in the case of Korea, null is not rejected only when $\alpha = 0.50$. Japan, Taiwan and UK show a difference parameter $d$ that is less than 0.5 for all $\alpha$ values. For Singapore, Thailand and the US, the difference parameter is less than 0.5 when $\alpha = 0.5$ or $\alpha = 0.55$, but greater than 0.5 when $\alpha = 0.6$. For the remaining countries (Australia, Canada and Korea), it is greater than 0.5 for all values of $\alpha$. Results from the GPH test have a few important implications. First, all time-varying exchange rate exposure beta series consistently reject both hypotheses $I(0)$ and $I(1)$. It suggests that all exposure betas in the sample are characterized by a $I(d)$ process with $0 < d < 1$ and can be recognized as long-memory or ARFIMA processes. Second, exposure betas for Japan, Taiwan and UK are covariance stationary as well as mean-reverting while exposure betas for Singapore, Thailand and the US are more likely to be so. However, exposure betas for Australia, Canada and Korea show covariance non-stationary, but mean-reverting dynamics.

The fact that the difference parameter $d$ for each exposure beta series is greater than 0 and less than 1 implies that they are mean-reverting, a feature that is also reflected in Figure 1. However, the impact of a shock on exposure betas is likely to decay hyperbolically, which is much slower than a rapid geometric decay represented by a standard ARMA process. Mean-reverting exchange rate exposure betas have both theoretical and empirical implications. First, since returns are linear functions of betas (that represent exposure to market risk, currency risk or any other risk), it is argued that mean reverting betas is an essential element in assuring the stationarity of returns. Second, the absence of mean reversion makes the notion of equilibrium have little relevance even in the long-run (Lai, 1997). Third, mean reverting exposure betas imply that these coefficients can be used for forecasting purposes. This may be extremely important news in hedging against currency risk.

### Table 4: GPH test results for time-varying exchange rate exposure betas

<table>
<thead>
<tr>
<th>Country</th>
<th>Value of difference parameter $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha = 0.50$</td>
</tr>
<tr>
<td>Australia</td>
<td>0.6267*</td>
</tr>
<tr>
<td></td>
<td>(-4.54)</td>
</tr>
<tr>
<td>Canada</td>
<td>0.7234*</td>
</tr>
<tr>
<td></td>
<td>(-2.33)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.1607*</td>
</tr>
<tr>
<td></td>
<td>(-5.43)</td>
</tr>
<tr>
<td>Korea</td>
<td>0.8270</td>
</tr>
<tr>
<td></td>
<td>(-1.34)</td>
</tr>
<tr>
<td>Singapore</td>
<td>0.3463*</td>
</tr>
<tr>
<td></td>
<td>(-6.34)</td>
</tr>
<tr>
<td>Taiwan</td>
<td>0.1445*</td>
</tr>
<tr>
<td></td>
<td>(-8.50)</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.3621*</td>
</tr>
<tr>
<td></td>
<td>(-5.81)</td>
</tr>
<tr>
<td>UK</td>
<td>0.2825*</td>
</tr>
<tr>
<td></td>
<td>(-4.45)</td>
</tr>
<tr>
<td>US</td>
<td>0.3145*</td>
</tr>
<tr>
<td></td>
<td>(-6.92)</td>
</tr>
</tbody>
</table>

Notes: $d$ refers to the difference parameter in the fractional integration process $\Phi(L)(1-L)^d Y_t = c + \Theta(L) u_t$, and is represented by $\phi$ in the regression equation $\ln I(\omega, j) = c - \phi \ln (4 \sin^2 (\omega, j/2)) + \zeta$; $t$-statistics are in parentheses; * indicates the significance at least at the 5% level.

### 6 CONCLUDING REMARKS

We have used a trivariate BEKK-GARCH(1,2,1)-M model based on a conditional ICAPM framework to obtain time-varying exchange rate exposure betas. Our approach does not require some prior understanding of the determinants of the time-variation of exposure beta to obtain the estimates of the same. Also, the suggested approach is more appropriate than the GARCH-based methods that use inappropriate mean structures in deriving time-varying betas. As the mean structure in such models does not represent a relevant ICAPM, the resultant information loss may lead to inaccurate estimates.

In deriving time-varying exchange rate exposure betas, we emphasize the necessity of taking the non-orthogonality between exchange rate changes and market returns into account. A portion of exchange rate exposure is always captured by market beta and is also priced under the label of market risk. What counts for the decisions of firms and investors is the portion of exchange rate exposure that is not captured by market beta and hence not priced under the market risk.
such, the estimated time-varying exchange rate exposure betas are more reliable than those in the studies that does not take this non-orthogonality into account.

The examination of the stochastic structure of the time-varying exchange rate exposure betas offers some useful insights. As evidenced by their basic statistics, irrespective of the nature of the market, exposure betas are usually more volatile than market betas. And, as compared to the market beta series for each economy, exposure beta series tend to have more outliers. Moreover, one can observe larger and more volatile exposure betas in emerging markets such as Korea, Taiwan and Thailand. Results from the GPH test reveal that exposure betas for all nine economies are long-memory processes characterized by fractional integration. Exposure beta in each case turns out to be mean-reverting, though their mean reverting dynamics could be highly persistent and display a slow hyperbolical decay. As for the covariance stationarity, however, we do not obtain unambiguous results and the matter is left for future research.

REFERENCES


